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Note on the Geometry of the Siderostat. By H. C. Plummer, M.A.

1. In the following note I have collected some of the principal geometrical propositions which relate to the siderostat and the motions connected with that instrument. There is little that can be considered essentially new in this account, but at the same time it may not be altogether useless to attempt to put the theory in a simple and clear light. Hitherto the siderostat has engaged the attention of the physicist rather than of the astronomer, and it is in France especially that its development has been promoted, while in this country it has been comparatively neglected. For these reasons there is perhaps some excuse for considering the matter in this place.

2. The principal propositions to which reference is made, and which are nearly all quite well known, are these :

1. The normal to the mirror describes a cone of the second order.
2. The circular sections of this cone are (*a*) perpendicular to the axis of the Earth, and (*b*) perpendicular to the ray reflected in a fixed direction.
3. The section (*a*) is described with uniform motion.
4. The section (*b*) is described with motion derived by stereographic projection from uniform motion.
5. The intersection of the mirror with the plane of reflexion describes a cone of the second order.
6. The circular sections of this cone are (*c*) perpendicular to the axis of the Earth, and (*d*) perpendicular to the ray reflected in a fixed direction.
7. The section (*c*) is described with uniform motion.
8. The rotation of the field is precisely the same as the motion with which the section (*b*) is described.

3. In the figure (fig. 1) which illustrates this note all directions are referred to a sphere of which the centre is C, the fixed centre of the siderostat mirror. The instrument is so adjusted that the star S, whose N.P.D. is δ, is reflected in the fixed direction CA. Hence the normal to the mirror CN bisects the arc SA. The pole is P, and is reflected to Q, so that the arc PQ is also bisected at N. It is at once obvious that AQ is equal to PS, and that Q therefore describes a circle round A of angular radius δ. The lines ACA′, PCP′ are diameters of the sphere. The intersection of the mirror with the plane of reflexion is represented by CM. This line is perpendicular to CN, and lies in the plane ASA′, which is clearly the plane of reflexion.

4. All the geometrical properties enumerated above become evident when the figure is examined. The lines A′S and CN are parallel for $SA'A = \frac{1}{2}SCA = NCA$. Similarly P′Q and CN are parallel for $QP'P = \frac{1}{2}QCP = NCP$. Hence the three

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lines A′S, P′Q, and CN, being parallel, generate cones which are similar and similarly placed. It follows that parallel sections of all three cones are similar. But the cone whose vertex is at A′ has a circular section which is uniformly described in a plane perpendicular to CP, and the cone whose vertex is at P′ has a circular section in a plane perpendicular to CA. Hence sections of the cones which are parallel to either of these planes are circular. Now the section perpendicular to A′A of the cone

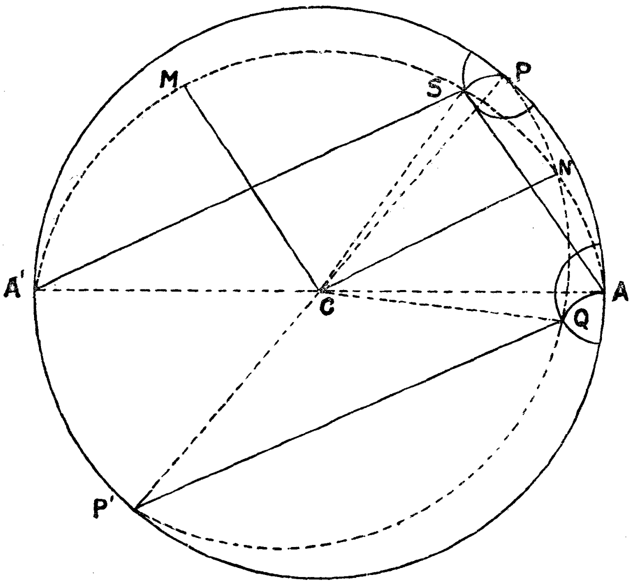

Fig. 1.

whose vertex is at A′ is the stereographic projection of the locus of S, which is the other circular section. We have thus obtained incidentally a fairly simple proof of the well-known theorem that the stereographic projection of a circle is a circle. But we have at the same time proved the propositions in § 2 numbered 4 and 8 as well as those numbered 1–3. The last proposition on the list is, as far as I know, new, and was obtained analytically before the possibility of so simple a geometrical proof was realised.

5. The remaining theorems refer to the motion of CM, and are derived with equal facility. For since CN is parallel to A′S, and CM is perpendicular to CN, CM is parallel to AS. Hence CM and AS generate cones which are similar and similarly placed, and the circular sections of the one are made by the same planes as those of the other. But the cone whose vertex is at A has for its sections the locus of S and the stereographic projection from A of this locus. The theorems 5–7 of § 2 follow at once.

6. The connection between the rotation of the field and the motion of the normal to the siderostat mirror suggests the possibility of deriving the mechanism for giving compensating motion to a photographic plate from the siderostat itself. The convenience

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and advantage of such an arrangement are obvious, but there are also difficulties to be anticipated. If it proved that these could be overcome, an interesting solution of the problem of compensation would result. It may be pointed out that in the stereographic projection from A′ of the small circle which is the path of S, the projection of P is not of course the centre, but corresponds to the point B (fig. 1, p. 402), which represents the position of the fixed pin employed in the method I have suggested recently for compensating the rotation. Hence we have this theorem in stereographic projection: The plane A′SP cuts the circle S again in S′. The projections of S and S′ are S_1 and S_1'. If the circle S is described uniformly, S_1' describes the projected circle uniformly, while the motion of S_1 obeys the law

$$\tan \tfrac{1}{2}\theta = K \tan \tfrac{1}{2} nt$$

7. The quite simple description of the geometry of the siderostat given above will render clear the principles which underlie the construction of this instrument. In Silbermann's device CS and CA are produced backwards and materialised as bars, and a linkage is added to form a rhombus with the diagonal of which coincides a slotted bar which is normal to the mirror. In all other devices the properties given in § 2, 3, or 7 are employed, and accordingly two types have resulted. In the one class we have the mirror driven by a rod fixed normal to the mirror; in the other by a rod or bar fixed in the plane of the mirror. The former type seems to have been first realised by S'Gravesande, and may be considered by imagining the mirror at A′ instead of C. Then A′S will represent the direction of the normal, which in its material form must of course be reversed, as A′T (fig. 2). It is only necessary to make T describe a small circle about the polar axis RC′ on a sphere which passes through the centre of the mirror. The point T represents a ring through which the normal passes, and which can be set at the appropriate polar distance. The second type of instrument was devised by Gambey. If we imagine the mirror transferred to A,

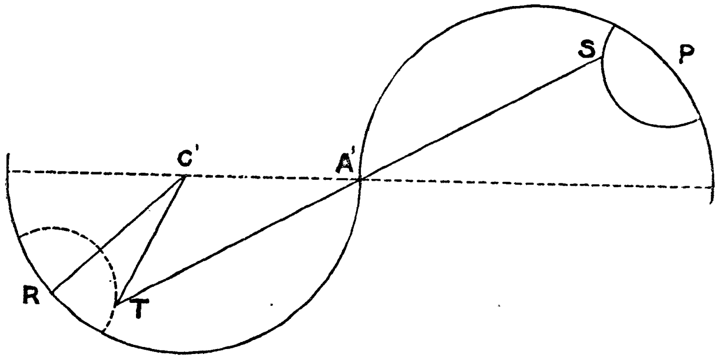

Fig. 2.

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it is easy to see, as in the other case, how the necessary motion is given to AS, the line in the plane of the mirror. But further the plane of the mirror must remain perpendicular to the plane ASA′. To secure this, that central line of the mirror which is perpendicular to the driving rod is constrained to remain perpendicular to CA, the direction of the fixed reflected ray. Thus the plane of the mirror contains a line which is perpendicular to two lines in the plane of reflexion, and the required condition is fulfilled. A small instrument, which has been found very useful, has been designed by Dr. Johnstone Stoney on this principle. The siderostat of Foucault combines some of the features of both types, the motion of the mirror being controlled partly by the normal rod and partly by a slotted bar in the plane of the mirror and passing over a rod which is the prolongation of TC′ (fig. 2). The bar is thus kept parallel to AS (fig. 1), as is easily seen, and provides the means of maintaining automatically the longer dimension of the mirror (if this is neither circular nor square) in the plane of reflexion.

The description of these instruments is of course too brief to convey a very satisfactory idea of the details of their construction. What is aimed at is to point out the relations of the different types. The principles outlined here will probably suffice to render the mechanism of any particular instrument immediately intelligible on inspection of the concrete example.

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The Spectrum of Nova Persei. Note 4.

By the Rev. Walter Sidgreaves, S.J.

In note 3 on the spectrum of *Nova Persei* in the last number of *Monthly Notices* it was observed that at the times of minimum of the light curve the bright Hζ band was greatly extended on its violet side, and that one of the blue bands appeared to be too bright to be satisfactorily explained by the greater contrast of a weaker continuous spectrum.

The photograph of March 28 shows two prominent lines of greater brightness in the so-called extension: one at its red side adjoining, or perhaps overlapping, the hydrogen band ζ, and the other near its margin on the more refrangible side. The wavelengths of these two lines are found to be 3882 and 3859, referred to the centre of bright Hβ. The extension may therefore be a cyanogen band, composed of the nineteen lines measured by Lockyer * between 3883 and 3855, of which the strongest lines are 3883, 3871, 3862, and 3855; and it will be convenient to call it, provisionally, the CN band. This band and the most prominent of the blue bands at 4633 are the characteristic features of the spectrum at the minima of the light curve. Both bands either

* Watts, Index of Spectra 168, revised edition.